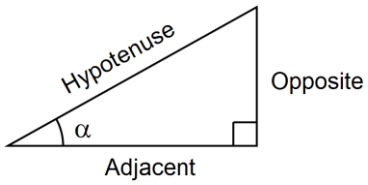
6.3 Notes Trigonometric Functions & Special Right Triangles

The six trigonometric functions are the sine (sin), cosine (cos), tangent (tan), cosecant (csc), secant (sec), and cotangent (cot) functions. There are several ways to define these functions of trigonometry. One of the most common mnemonic devices is SOH-CAH-TOA.



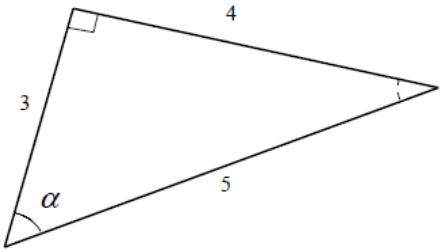
$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc\alpha = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\alpha = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\alpha = \frac{\text{adjacent}}{\text{opposite}}$$

Reciprocal Identities: $\csc\alpha = \frac{1}{\sin\alpha}$ $\sec\alpha = \frac{1}{\cos\alpha}$ $\cot\alpha = \frac{1}{\tan\alpha}$

Examples:

Use the following triangle to evaluate $\sin(\alpha), \cos(\alpha), \tan(\alpha), \csc(\alpha), \sec(\alpha),$ **and** $\cot(\alpha)$.

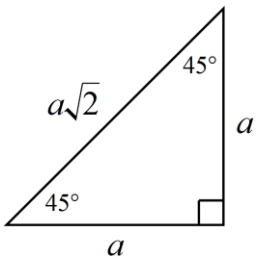


Evaluate the remaining trigonometric ratios using the given trigonometric ratio.

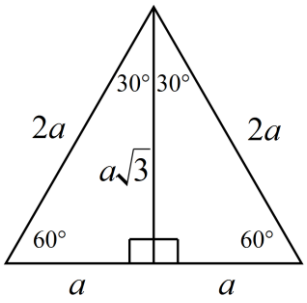
$\sin\theta = \frac{2}{5}$

$\cot\theta = \frac{6}{5}$

Ratios for Special Right Triangles



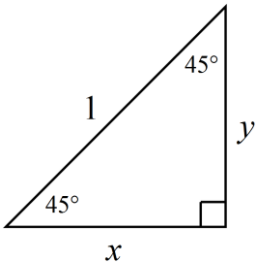
hypotenuse = leg $\cdot \sqrt{2}$

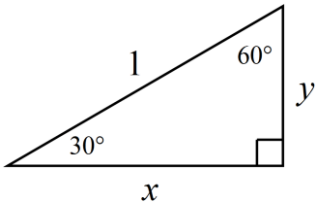


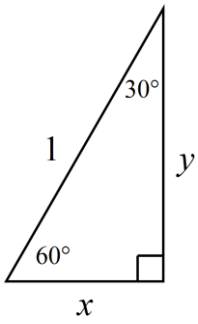
hypotenuse = $2 \cdot$ short leg

long leg = short leg $\cdot \sqrt{3}$

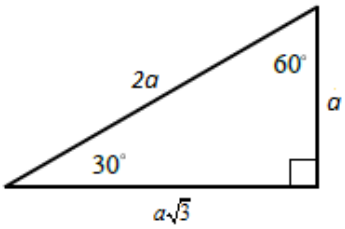
Find the values of x and y in the triangles below:

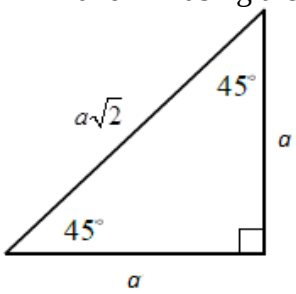






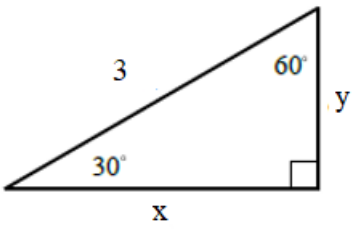
Find the values of the six trigonometric functions for 30° and 45° using the triangles below.

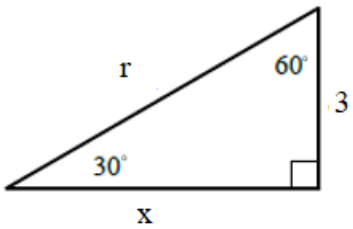


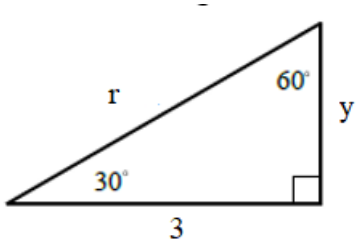


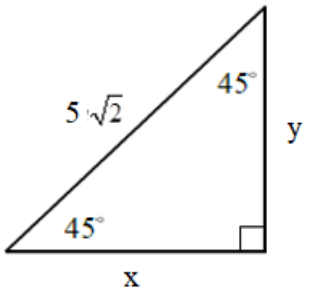
Examples:

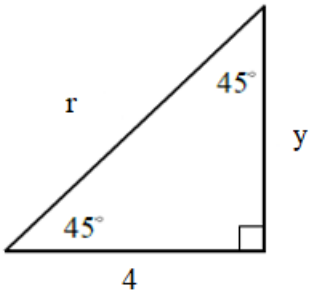
Find the missing sides using the special right triangles.

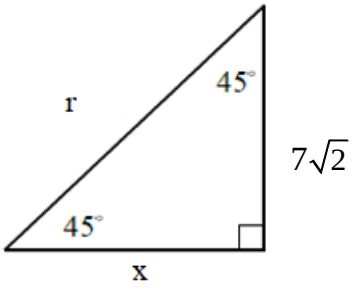










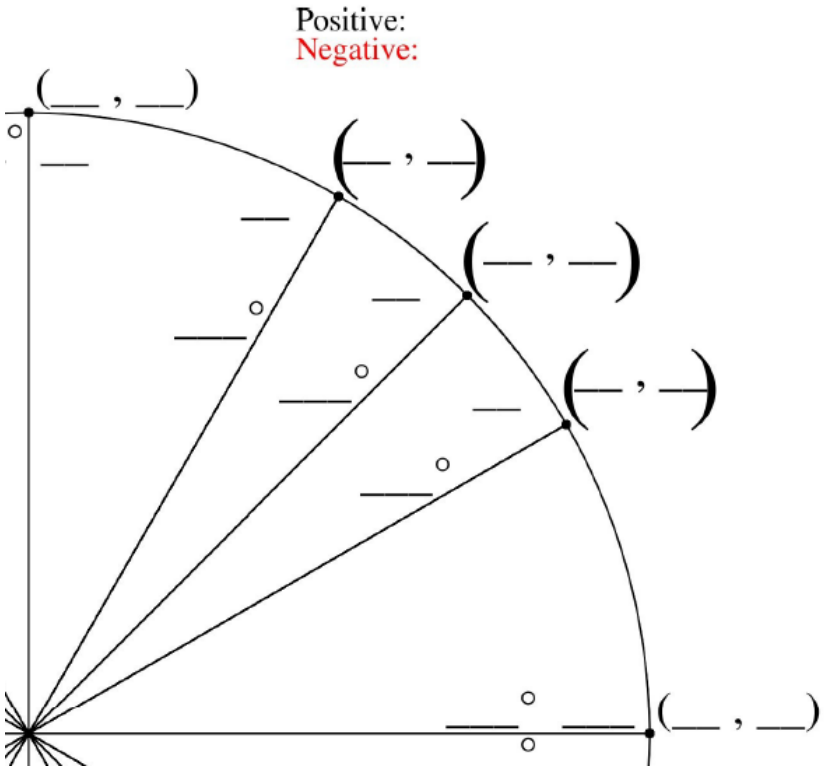


After rationalizing denominators and adding trigonometric functions for 60°, we summarize the trigonometric function values for these special cases in the following table.

Trigonometric Function Values for 30°, 45° and 60°

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Let's build the unit circle in the first quadrant using the special right triangles!



Examples:

Find the exact values without using a calculator.

$\sin\frac{\pi}{6} =$ $\cos\frac{\pi}{2} =$ $\tan 45° =$ $\cot 60° =$ $\sec 0 =$

Find the given values using a calculator. Check the mode! Round answers to the nearest ten thousandths.

$\sin 5° =$ $\csc 51° =$ $\tan \frac{\pi}{5} =$ $\cos 0.786 =$

Find the acute angle that satisfies the given equation. Give angle in both degrees and radians without using a calculator. Hint: Use the unit circle we filled out.

$\cos\theta = \frac{\sqrt{3}}{2}$ $\sin^{-1}\frac{\sqrt{2}}{2} = \theta$ $\tan\theta = \textit{undefined}$

Find the acute angle that satisfies the given equation using a calculator. Check the mode. Round answers to the nearest tenth of a degree.

$\cos\theta = 0.8632$ $\sin^{-1}(0.1324) = \theta$

Solve the triangle using the given parts.